Solution Sketches

CS@Mines High School Programming Competition 2023

Saturday, April 29th, 2023

Colorado School of Mines

Above Sea Level (Ryan Mapes)

Summary

Calculate the modified elevation of a particular point after the sea level rises by 30 centimeters.

The solution is to read the input (which is given in meters) as a floating point number, subtract 0.3, and output it.

You need to subtract because as the sea level *rises*, the **elevation** of all objects on land decreases.

Driving Dilemma (Sumner Evans)

Summary

Will Rishi make it to the end of the intersection before the light turns red given he is going at S miles per hour? He is D feet from the end of the intersection and he has T seconds to get through the intersection.

For this problem, you need to determine if Rishi will travel at least D feet in T seconds going at S miles per hour.

You need to do some unit conversions in order to make this comparison. It is recommended to convert the speed to feet per second.

Then if $T \times S \geq D$ Rishi "MADE IT" otherwise output "FAILED TEST".

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MIR Cipher (Ryan Manley)

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Given a message, decode it by applying a Cæsar cipher to each character with shift amount that doubles every character.

For each character, shift it by the shift amount, print it, then double the shift amount. Main gotchas:

1. The shift amount will quickly become greater than the length of the alphabet so you will need to wrap around by doing all operations under $\pmod{26}$.
2. You need to use 64-bit integer to store the shift amount!

It is recommended to subtract from the ASCII value of the letters before performing the Cæsar cipher so that A is 0, B is 1, etc. This also allows the modular arithmetic work naturally.

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Sarah’s Sandwich Shop (Ethan Richards)

Summary

Given a word, determine what numbers on a standard keypad would be used to represent that word.

You need to convert everything to lower case (or upper case), then loop over all of the characters in the string and determine the number corresponding to the letter.

One of the cleanest ways to accomplish this is by creating a *dictionary* to store a mapping of letters to numbers.

Alternatively, you can just write 26 `switch` cases or `if` statements.

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Mines Football (Ethan Richards)

Summary

Calculate the maximum and minimum number of points scored across all games, and the maximum and minimum total number of points scored in a month.

Store the current maximum score, current minimum score, current maximum total month, and current minimum total month (minimum values should be initialized to `INT_MAX`).

Loop through the scores for each month to sum up the scores for the month. Use that to update the current minimum and maximum total month scores.

Either in the same loop or in a separate loop, iterate through each of the individual scores in the month and update the current maximum and minimum scores.

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Cookie Monster Concussion (Scott Enriquez)

Summary

Compute the output of an algorithm that determines if an integer is divisible by nine.

The problem can be solved by implementing the algorithm provided in the problem statement. It may have been useful to interpret the input as a string of digits rather than an integer to avoid overflow on 32-bit integers.

Note that running the provided algorithm on a number C is equivalent to computing $C \mod 9$, except that the algorithm outputs 9 if $C \mod 9 = 0$

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Compass Rose (John Henke)

Summary

Given a list of headings in a generalized extended version of the compass rose cardinal directions, determine the corresponding degree values.

First, handle the special cases of due north/east/south/west.

Then, the easiest way to solve this is by dividing the problem up per-quadrant.

You can determine which quadrant of the compass rose you are dealing with looking at the last two characters of the heading.

Once you know the quadrant, the preceeding characters (going right to left) add specificity to the heading.

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Compass Rose: Adding Specificity to Headings

The most intuitive way to think about how each subsequent preceeding character adds specificity is in terms of a *binary search*.

Each character “pulls” the bounds towards that cardinal direction.

For example, if you have the heading NE (45°) it can be thought of as the mid-point between 0° (N) and 90° (E).

If we prepend N to NE to get NNE, we “pull” the bounds *towards* the north, and we get the mid-point between 0° (N) and 45° (NE) which is 22.5°.

There is no great way to do this entirely generically for all four quadrants, and some amount of special-casing per quadrant will be necessary.

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Computer Imaging (Colin Siles)

Summary

Given many different flashdrives that can image computers at different speeds, and the number of computers to image, determine the minimum time to image all computers

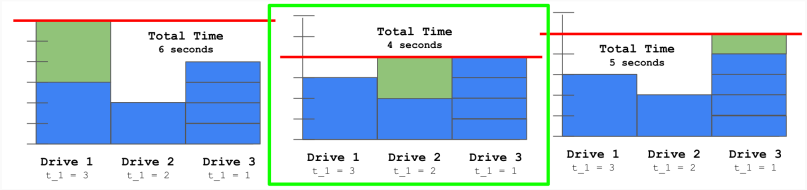
It's useful to think about this problem as a scheduling problem, where computers are scheduled onto flash drives. The input size for this problem is small enough that a greedy simulation is fast enough.

- Keep track of the total amount of time each flash drive images a computer (initially zero for each drive).
- For each computer we need to image, we determine which flash drive it should be scheduled on.

Computer Imaging: Greedy Simulation

We choose the flash drive based by determining which one will **minimize the total time**. To do this, we iterate over each flash drive and consider the total time if we scheduled the next computer on that flash drive.

We select the flash drive that resulted in the lowest total time (choosing arbitrarily if multiple options yield the same minimum time), and increment the total amount of time that flash drive images a computer accordingly.



Computer Imaging: Final Steps

After we complete this procedure for each computer, the maximum amount of time that any flash drive is imaging a computer is the minimum amount of time it would take to image all computers.

Scheduling any computer on a different flash drive would necessarily increase the total time (or at least leave it unchanged), per the nature of the greedy simulation.

If N is the number of computers, and M is the number of flash drives, this solution is $\mathcal{O}(NM)$. This is fast enough since both N and M are less than or equal to 1 000.

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Computer Imaging: Alternative Solution

The problem can also be solved by doing a linear search of the total time, starting from 0, to determine the smallest time when the required number of computers are imaged. If there are M flash drives, and the ith flash drive takes t_i seconds to image a computer, then the total number of computers that can be imaged in T seconds is

$$\sum_{i=1}^{M} \left\lfloor \frac{T}{t_i} \right\rfloor .$$

Note that there are more efficient solutions, which are required for the more difficult form of this problem.

Planetary Computer Imaging (Colin Siles)

Summary

Given many different flash drives that image computers at different speeds, and a very large number of computers to image, determine the minimum time to image all computers.

In this version of the problem, the number of computers can be very large (up to $8 \cdot 10^9$), so neither the greedy simulation nor the linear search is fast enough.

We can use the same formula from the linear search solution to determine how many computers are imaged in a given amount of time, but use **binary search** to drastically reduce the number of times we search, compared to linear search.

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Planetary Computer Imaging: Binary Search

Since each step of binary search cuts the number of possible candidates in half, we can determine the answer amongst the $2 \cdot 10^{11}$ possible times in only $log_2(2 \cdot 10^{11}) \approx 38$ steps!

If we test a time and image more computers than necessary, then we can eliminate all times greater than that value, since all such times must necessarily image at least as many computers.

If we test a time and image fewer computers than necessary, then we can eliminate all times less than that value, since all such times could only possibly image fewer computers.

Planetary Computer Imaging: Alternative Solution

A theroetically optimal schedule would be one where every flash drive is being used throughout the entire process, and all flash drives finish imaging the computers at the same time. This is impossible to achieve in general since we cannot image a computer with multiple flashdrives, but it lends a useful insight.

Assuming we could partially image a computer, in time T, we could image $N = \sum_i \frac{T}{t_i}$ computers. Rearranging, we obtain $T = \frac{N}{\sum_i 1/t_i}$, where T is the "optimal" time to image N computers. If we find the integer number of computers that can be imaged by each flash drive in that time for the given input, we will only need to schedule $\mathcal{O}(M)$ more computers. A greedy simulation for those final computers would be $\mathcal{O}(M^2)$, which is fast enough.

Fixing Figures (Ethan Richards)

Summary

Convert a number to its textual representation.

The cleanest approach is to notice that the textual representation of each set of 3-digits is the same, with an added postfix for thousands/millions.

With this insight, you can solve printing for $[0, 999]$ and then use that solution for the hundreds place, the thousands place, and the millions place.

There are some highly annoying edge cases such as numbers from $[0, 19]$.

There are also additional annoying cases such as double-digit numbers with hyphens.

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Paper Pile Pandemonium (Colin Siles)

Summary

Given the initial state of a series of stacks of paper, and a record of how sheets of paper were moved between stacks, determine the final state of the stacks

To solve this problem, simulate the provided sequence of operations on the initial state of the stacks, and then output the final state.

The simulation must keep the sheets in order when they were moved between piles, and not reverse their order.

The size of the inputs is small enough that no special data structures are required: dynamic arrays to represent each stack is sufficient.

Noise Reduction (Sumner Evans)

Summary

Given a list of N temperature readings, find a group size such that the difference of averages between sequential groups is below a certain threshold, T.

The bounds of this problem are such that you can just try every group size up to $N/2 + 1$ and see if the group size works.

In order to determine if a group size g works, split the dataset into groups of size g (discarding any extraneous readings) and compute the average of the readings within each group.

Keep track of the previous group's average, and if the difference is above T, then the group size is too small.

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Telescope Targeting (Sam Sartor)

Summary

Given a $W \times H$ reference image and an $N \times M$ sky, determine the **rotation** at which the reference image would be centered in the sky.

To find where the reference image appears in the current view, create 4 nested for loops:

1. Over $M - H + 1$ possible vertical offsets
2. Over $N - W + 1$ possible horizontal offsets
3. Over H rows of pixels in the reference image
4. Over W pixels in the row of the reference image

If all pixels in the reference image equal the pixels in the current view, at the given offset, then the correct offset has been found.

Telescope Targeting: Calculating Rotation

However, the offset itself is not the answer, we need to calculate the rotation needed to center the reference image.

The horizontal rotation is horizontal offset $- \frac{N-W}{2}$

The vertical rotation is vertical offset $- \frac{M-H}{2}$

Wikipedia Black Hole (Sumner Evans)

Summary

Given a list of links between Wikipedia pages, determine the smallest number of clicks it would take to get back to the page you started on.

Model the problem as a **directed graph** with links being the *edges* and pages being the *nodes*.

Because you need to output the *smallest* loop, you must use a **breadth-first search** (BFS) to find the loop rather than a depth-first search (DFS).

If you explore the entire graph without finding a loop back to the original page, then there is "NO BLACK HOLE".

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Unit Rescue (Alex Capehart)

Summary

Given a set of conversion factors between units, convert from one unit to another unit. (There may not be a direct conversion given.)

Model the problem as a **directed graph** with the *nodes* being units and the *edges* being a known conversion from the two units. Note that you will always have the reverse edge since you can always invert the ratio.

Then, use BFS or DFS to find a path from the start unit to the end unit. In addition to keeping track of which nodes to visit, you will have to track the converted value (similar to a distance table used in Dijkstra's algorithm).

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Basketball Modeling (Colin Siles)

Summary

Given a probabilistic model for how a basketball team scores points, determine the expected value for the number of points they will score.

This problem requires a solution technique called "Dynamic Programming", which allows us to efficiently solve problems involving recursion.

Let $f(n, m_2, m_3)$ be the expected number of points the team scores over n posessions given that they begin with an m_2 percent chance of making a 2-pointer, and an m_3 percent chance of making a 3-pointer. We define the function recursively.

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Basketball Modeling: Recursive Definition

For the base case, $f(0, m_2, m_3) = 0$ for all m_2 and m_3, because there are no more posessions to score points.

For the recursive case, let C_2 and C_3 be the confidence adjustments. There are five cases to consider.

If the team attempts and makes a 2-pointer, the expected number of points scored is $2 + f(n-1, min(m_2 + C_2, 100), m_3)$. The team has scored 2 points, and we add the expected number of points they score over the remaining possessions.

If the team attempts but misses a 2-pointer, the expected number of points scored is $0 + f(n-1, max(m_2 - C_2, 0), m_3)$. The team did not score any points, but we add the expected value of the rest of the possesssions.

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Basketball Modeling: Final Steps

The 3-pointers and no shot attempt cases follow a similar pattern.

To complete the recursive definition, we multiply the expected value of each case by the probability of that case occuring, and sum those values up. The probability of attempting a 2-pointer and making it is $\frac{A_2}{100} \cdot \frac{m_2}{100}$, for example.

Computing this function with the recursive definition directly is not fast enough. Because all of the inputs to f are integers, we would repeat a lot of computations. If we “memoize” the results to the function call (that is, cache the answer, and check the cache before computing), we can solve the problem efficiently. This memoization step is what advances us from simple recursion to “Dynamic Programming”.

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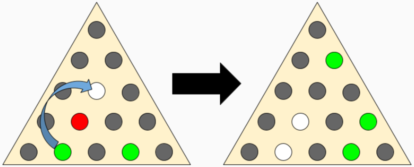
Summary

Given the state of a Peg Game, determine the number of pegs that you would end up with after optimal play.



There are three main challenges to solve this problem:

1. Modelling the problem as a graph
2. Representing the state of the board in a comparable way.
3. Finding adjacencies to a given hole.

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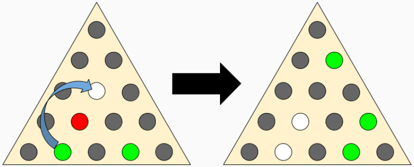

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2. Representing the state of the board in a comparable way.
3. Finding adjacencies to a given hole.

Pegs: Modelling

This problem requires that you model the possible moves from a given position as *edges* in a directed graph. The *nodes* of your graph represent a game state.

By modelling the problem in this way, you can use an exhaustive search (either BFS or DFS work) to find the game state with the smallest number of pegs remaining.

Pegs: Representing the board State

There are many ways to represent the state of the board, but whichever way you choose, it must be hashable so that you can put it into a “visited” set so you don’t get in an infinite loop.

Options include:

- **Bitsets**: there are only 15 pegs, so you can fit the entire board state in a 32-bit integer (or even a 16-bit one).
- **Tuples**: if your language has tuples (say, Python), you can use tuples of booleans.
- **Custom class with hash function**: you may have to resort to this in languages such as Java.

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Pegs: Finding Adjacencies

Finding adjacencies to a given hole is nontrivial due to the triangular nature of the board.

Given a hole in the (r, c) cell, it is adjacent to the holes at the $(r, c-1)$, $(r, c+1)$, $(r-1, c-1)$, $(r-1, c)$, $(r+1, c)$, and $(r+1, c+1)$.

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Galactic Reconstruction (Kelly Dance)

Summary

Given a starting set of *n* colonies, and a set of proposed warp gates, determine which gates will be built, which are unnecessary, and which are impossible to build due to lack of funds.

This problem requires that we efficiently keep track of *disjoint subsets* of the colonies.

This problem can be solved using the *Union Find* data structure, but we will go over another solution that does not require as much prerequisite knowledge.

Galactic Reconstruction: Set Merging Initialization

We will represent each cluster as a tuple of the set of colonies within it and its wealth. We will also need a list that tracks with cluster each colony is part of. So before we process any propositions, we have these two structures:

`clusters`

$$[(\{1\}, w_1), (\{2\}, w_2), (\{3\}, 3), \ldots]$$

`lookup`

$$[1, 2, 3, \ldots]$$

Galactic Reconstruction: Set Merging Proposition Processing

We now process the propositions in order. If the current proposition is joining *a*,*b* at at a cost of *c*, we can check `lookup[a]` and `lookup[b]` to see which clusters they are part of.

If they already belong to the same cluster then you can just output that the warp gate is `UNNECESSARY`.

We can then check that each have enough wealth to build the warp gate using `clusters[lookup[a]][1]` and `clusters[lookup[b]][1]`. If either has wealth less than *c*, then it is `IMPOSSIBLE`.

If both of these checks have passed, we output `BUILT` and must update our data structures to reflect this change.

Galactic Reconstruction: Set Merging Proposition Processing

We now process the propositions in order. If the current proposition is joining *a*,*b* at at a cost of *c*, we can check `lookup[`*a*`]` and `lookup[`*b*`]` to see which clusters they are part of.

If they already belong to the same cluster then you can just output that the warp gate is `UNNECESSARY`.

We can then check that each have enough wealth to build the warp gate using `clusters[lookup[`*a*`]][1]` and `clusters[lookup[`*b*`]][1]`. If either has wealth less than *c*, then it is `IMPOSSIBLE`.

If both of these checks have passed, we output `BUILT` and must update our data structures to reflect this change.

Galactic Reconstruction: Set Merging Data Structure Updates

To update our data structures, we choose the *smaller* cluster and move its elements into the larger cluster. Moving a colony to the larger cluster requires that we

1. add the colony to the larger cluster
2. update the lookup table for each of the moved colonies

We must also update the wealth of the larger cluster, which now represents their union, to be the sum of the original wealths minus twice the warp gate cost.

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Galactic Reconstruction: Set Merging Complexity Analysis

This algorithm is $\mathcal{O}(n \log n)$. The most expensive part of this algorithm is moving colonies from one cluster to another.

We can derive our complexity by looking at how many times a colony can move from one cluster to another then multiplying that count by the number of colonies (n).

Since we are always moving a colony into a cluster with a size greater than or equal to the size of the previous cluster, we know that the size of the cluster a colony is part of *at least doubles* after every move.

Since the size is doubling, we know that there can be at most around $\log_2(n)$ moves before there is only a single cluster.

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